Technical Note

Title: Masonry – Advanced Yield Line Analysis Method

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Program: Masonry Wall Design

MasterKey Masonry Design: Advanced Yield Line Analysis

The MasterSeries Masonry module provides a yield line based analysis in addition to the use of the traditional analysis methods based on bending moment coefficients to give the distribution of the bending moments in rectilinear panels supported on 3 or 4 sides. The advantage of the Advanced Yield Line method lies in the ability to deal with more complex panel geometries which enables the analysis and design of panels with more complex arrangements of openings or support conditions.

Traditional Masonry design methods

Both the British Stand and Eurocode (UK National Annex) base the design of walls subject to lateral loads on the assessment of resistance of the wall in bending against the applied moments due to the lateral load, with an enhancement of the bending resistance under vertical loads being taken into account. For walls supported on one side or on two opposite sided, the maximum bending moment can be determined by use of standard formulae which relate the wall dimensions and loading to the point of maximum bending moment.

In the case of walls supported on 3 or 4 sides, the Codes provide tabulated values of bending moment coefficients which relate the wall length, height and the orthogonal ratio of the flexural strength of the masonry for the parallel and perpendicular directions.

However, in both the British Standard and Eurocode, the use of a "recognized method of obtaining bending moments in flat plate e.g. yield line or finite element…" is permitted. As such, the yield line method is permissible under both the British Standard and Eurocode.

The tabulated values for the bending moment coefficients are derived from the yield line analysis method, taking into account the material anisotropy. Therefore, while it may appear that the code procedures are distinct from the yield line analysis, they are in fact derived from the same analysis methodology.

For walls with openings, the British Standard provides some guidance on a method to treat the subpanels of the wall around the opening as individual wall panels. This approach is similar to the Hilleborg strip method developed for concrete slabs. The division into panels is an approximate

method and requires some experience to ensure that the subdivision is appropriate. For walls with openings designed using the Eurocode, the EC gives no guidance on the use subdivision of panels, making reference only to the yield line method for panels with substantial openings.

The Yield Line Analysis method

The yield line method was originally developed in in the 1940's and 50's by the Danish engineer K. W. Johansen and originally aimed at the analysis and design of reinforced concrete slabs. Subsequent research and testing through the 1960's to the 1980's confirmed the validity of the theory, with excellent agreement being obtained between the theoretical and experimental results, validating the applicability and effectiveness of the method.

The yield line method is a plastic analysis method which establishes the load at failure based on the assumed failure mechanism and yield line pattern. The yield line theory assumes that there is sufficient ductility in the section and material under consideration such that the failure mechanism can occur. While this assumption can easily be taken for concrete, masonry is a brittle material and at first it may not appear that an analysis method relying on sufficient ductility can be readily applied to masonry panels. However, as noted in the Concrete Centre publication "Practical Yield Line Design", testing of walls panels to failure demonstrated compatibility between the loads at failure and the failure loads predicted by theory.

Determining the failure mechanism

The yield line method is generally based on an energy method based on virtual work, although it is also possible to use an equilibrium method. The Masterseries Masonry module uses the principal work method. The method is based on determining a failure mechanism consisting of a pattern of yield lines which develop in a wall panel and then equating the work done by the lateral load and the work done by the rotation of the yield lines assuming plastic behaviour of the bending moments along the yield lines. The yield lines divide the surface into a series of sub-elements, bounded by yield lines or by the panel boundary. Each sub-element is assumed to remain planar, ensuring geometric compatibility along yield lines. Since the elastic deformations are small compared with the plastic deformations, this assumption is not critical to the accuracy of the results.

For a selected yield line pattern, the work done by the load and the work done by the rotation of the yield lines are calculated and, applying the Conservation of Energy principle, these values are equated. In the case of the Masterseries Masonry module, a factor is applied to the loading and the result is then calculated in terms of this factor. This then allows the calculation of the factor that the applied loading would need to be multiplied by to lead to failure on the assumed yield line pattern. This allows a utilization ratio to be calculated for the masonry panel.

The work done by the yield lines is, therefore, dependent upon the trial solution yield line pattern. The yield line method is an upper bound method; therefore, the selected yield line pattern either gives an exact solution for the collapse load, or the method overestimates the load required. As such, the accuracy of the method is dependent upon the yield line pattern; to identify the collapse mechanism it is necessary to identify the yield line pattern which gives the lowest collapse load.

In the case of standard geometry panels with no openings, it is a reasonably simple task to identify the critical yield line pattern. For rectilinear panels with orthotropic materials, symmetry will occur

in the yield line pattern at failure. In such a case, it is possible to define the yield line pattern involving a variable x, representing some distance in the pattern, and then to solve for x and, using calculus or a numerical method, identify the minimum failure load. However, where the geometry of the panel, due to edge conditions or the location of openings, means that symmetry cannot be taken, this method will not work, since the yield line pattern will become be more complex and unlikely to show any symmetry. Therefore, to use manual methods, trial solutions of yield line patterns will be required, but the resulting complexity of the yield line that would need to be tested means this approach could become very time intensive and impractical.

To overcome the issue of an upper bound type solution potentially over estimating the collapse load, the Masterseries Masonry module utilises an automated yield line algorithm, which refines the yield line pattern by an adaptive, iterative process. Thus, a yield line pattern is refined and the failure load calculated, and the process then repeated, until the critical yield line pattern is identified. The Masterseries Masonry module is, therefore, calculating multiple yield line patterns until the supremum, the lowest value of the upper bound solutions, is identified. To achieve this, the software uses an underlying mesh of points from which the yield lines are created. The lines between the points are potential yield lines and the final yield line pattern will be made up of some pattern of these potential lines. The mesh density can be changed within the Masonry module, to refine the density of points and establish the critical yield line pattern. However, with a finer mesh, the number of calculations required as part of the yield line analysis grows and the use of a very fine mesh can increase the calculation time. Where the utilisation ratio for the wall is greater than 0.95, that is where the wall is calculated to be greater than 95% of capacity, it recommended that the analysis be undertaken with a finer mesh density.

Analysis Validation

During the process of developing the Advanced Yield Line method, a process of validation of the results was undertaken. This involved comparing the outputs from the Masonry Module with hand calculations. Comparison was also done between the results taken from the yield line method and the traditional code based methods. However, these were used as a secondary test, given that the underlying method used to develop the code based methods was the yield line method.

Hand calculations were carried out on a range of rectilinear panels with a range of support conditions and loadings and the critical yield line patterns were determined by using first principles. This involved determining the yield line pattern in terms of a variable x, which represented some length determining the position of a point within the yield lines. The work done by the load and the yield lines was equated and then rewritten to give the work done in terms of the variable x. To determine the minimum, the expression was differentiated. This gives a polynomial expression which can be solved either directly, or by using numerical solutions. The critical yield line pattern was then compared using the Masonry module output for the yield lines.

The results of the validation testing gave a very strong correlation between the results of a first principle analysis and the software outputs.

Analysis Validation Example

The following test wall panel has been designed using the Yield Line analysis method. For ease of calculation, a single leaf wall panel with no openings has been considered. The material properties have been manually adjusted to give a moment capacity of 3 kNm/m in both the perpendicular and parallel directions. A lateral wind load of 1.5 kN/m² has been applied with a load factor of 1.5, to give an ultimate lateral load of 2.25 kN/m². The MasterSeries Masonry output is given below.

TWO WAY SPANNING, VERTICALLY AND LATERALLY LOADED, SINGLE-LEAF WALL

DESIGN TO BS EN 1996-1-1:2005
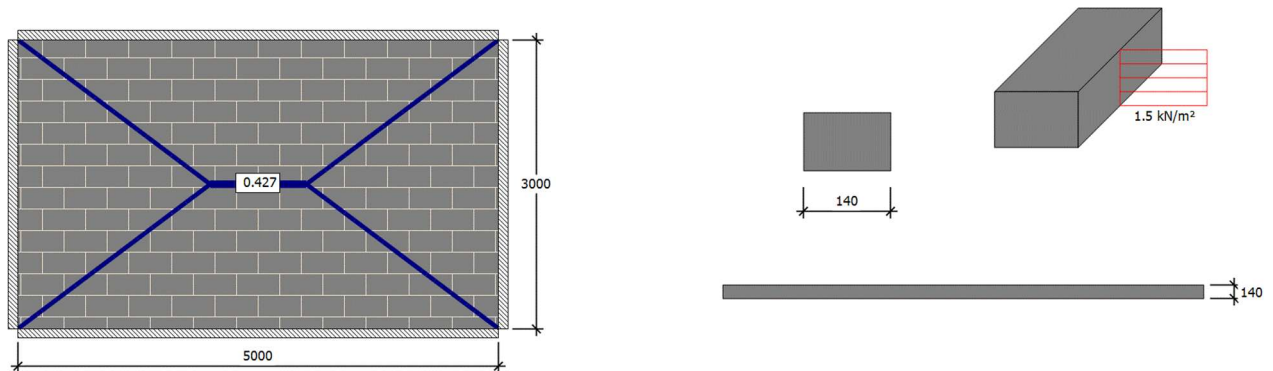
Summary of Design Data

EuroCode National Annex	Using UK values:A1 2012		
Wall Dimensions	h=3.000 m, hef=2.206 m (Eqn. 5.8), L=5.000 m, Lef=5.000 m		
Support Conditions	Bottom Simple, Top Simple, Left Simple, Right Simple		
Lateral Loads	Wx=1.5 kN/m²		
Single-leaf Wall (mm)	t=140, tef=140		
Limiting Dimensions	λ=15.8<=λlim=27, L/tef=35.7, H/tef=21.4, Hence H/tef<=62.9	0.584	OK

Wall Design

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Partial Safety Factor (γmc/γmf)	Construction Class 2, Unit Manufacture II	3/2.7	Table NA.1
Unit Material	Concrete Blocks, Group 1, γ=20 kN/m³		
	Normalised mean compressive strengthf_b =17.5 N/mm²		
Mortar Material	M4, fm =4 N/mm²		
Unit Ratio	Unit height=215, Least horizontal dimensions=100	2.15	
Compressive Strength (f_k)	k = 0.75, α = 0.7, β = 0.3	8.43 N/mm²	Table NA.4
Section Properties	Area=1400 cm²/m, Zp=3267 cm³/m		
Flexural Strength f_{xk2}(Perpendicular)		2.48 N/mm² from test	
Flexural Strength f_{xk1}(Parallel)	f_{xk1}=2.399, gd=0.03 N/mm²		
	$f_{xk1}=f_{xk1}$+min(gd, 0.2•f_k/γmc)γmf	2.48 N/mm² from test	
Critical axial compressive case	1.35(γ.tk.h)		
Max local stress @	X=0 m,Y=1.5 m < fk/γmc	0.08 N/mm²	OK
Critical axial buckling case	1.35(γ.tk.h)		
Max axial buckling force @	X=2.5 m, Y=1.5 m averaged over width of 1.4 m	11.34kN/m	
Moments from Lateral Load	$M_{wx,top}$=0.000 kN.m, $M_{wx,mid}$=0.000 kN.m		
Capacity reduction factor top, Φ	ex=0.0 mm, hef=2206 mm, tef=140.0 mm, t=140.0 mm	0.900	
Capacity reduction factor mid, $Φ_m$	Creep coef. =1.5, ehm = 0.000 mm, h_{ef} = 2.206	0.729	
Fr=Φ.f_k.tk/γmc	0.729x8.43x140/3	287.0 kN/m	
Fd/Fr	11.3/287.0	0.040	OK
Mr=f_{xk2}.Zp/γmf	2.48x3267/2.7	3.000 kN.m/m	
Mr=f_{xk1}.Zb/γmf	2.48x3267/2.7	3 kN.m/m	

Design for Lateral Loads

Design Lateral Load Wd	1.5 Wx	2.250 kN/m²	
Yield Line Analysis	Load Factor, $λ_p$	2.343	
Ut=1/$λ_p$	1 / 2.343	0.427	OK

Check Calculation on wall capacity
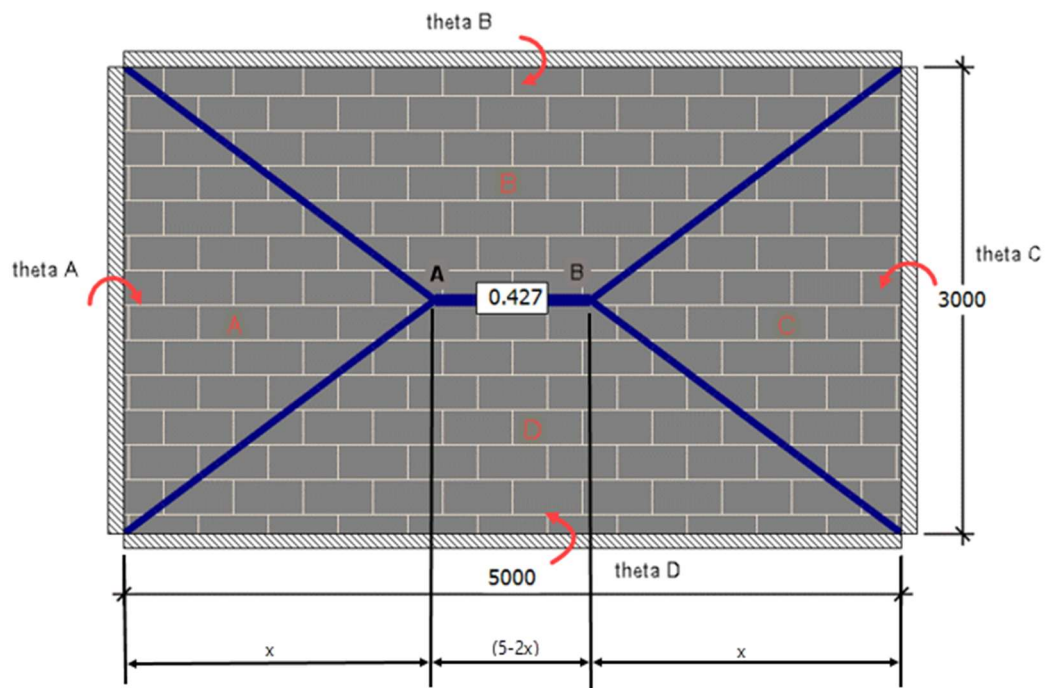
Load on wall **w** = 2.25 kN/m² (ultimate limit state)

Bending capacity of wall **m** = 3.0 kNm/m (perpendicular and parallel directions)

Wall rotations: $\theta_A = \theta_C = 1/x$ and $\theta_B = \theta_D = 1/1.5$ based on a displacement if 1 on the line **AB**

The virtual work (VW) work done by a yield line is given by

VW(m) = m * L * θ

Where L is the projected length of the yield line in the perpendicular or parallel direction.

Hence, the virtual work done by yield lines is:

$$VW(m) = 2 * \left(m * \frac{1}{x} * 3\right) + 2 * \left(m * \frac{1}{1.5} * 5\right)$$

$$= 2 * \left(3 * \frac{1}{x} * 3\right) + 2 * (3 * \frac{1}{1.5} * 5)$$

$$= \frac{18}{x} + 20$$

The virtual work done by the load is calculated using the yield lines to subdivide the wall panel into sub panels. The total work done is then the sum of the work done per sub-panel, where the work per sub-panel is equal to the total load on each sub panel multiplied by the displacement of the centroid of the load, taking the point or line of maximum displacement being equal to 1. Therefore, the work done by lateral load is given by

$$VW(w) = 2(w * x * 3 * \frac{1}{2} * \frac{1}{3}) + 2(w * (5 - 2x) * 1.5 * \frac{1}{2}) + 4(w * x * 1.5 * \frac{1}{2} * \frac{1}{3})$$

$$= \frac{15}{2} * w - wx$$

From the Masterseries output, the position of the yield lines is defined by the distance x, where x = 1.992m, and so

$$VW(w) = w\left(\frac{15}{2} - 1.992\right)$$

At failure, the work done by the yield lines and the work done by the load are equal. Hence, equating work done then gives:

$$VW(m) = VW(w)$$

And so

$$\frac{18}{1.992} + 20 = w\left(\frac{15}{2} - 1.992\right)$$

Hence the failure load based on the yield line pattern with x = 1.992m is

$$w = \left(\frac{18}{1.992} + 20\right) / \left(\frac{15}{2} - 1.992\right)$$

$$w = 5.271 \text{ kN/m}^2 \text{ (ultimate)}$$

The ratio of the applied load to the calculated failure load is then:

$$\frac{2.25}{5.271} = 0.426$$

The utilisation factor from the Masterseries Masonry calculation is 0.427.

References

During the development process the following publications were used. The primary theory used was based on Munro [1] et al. The limitations of the Munro method were overcome utilising the principals discussed in [2] and [3].

[1] Munro J, Da Fonseca A, 1978. Yield line method by finite elements and linear programming. Struct. Eng. 56B, 37–44

[2] Johnson, D. 1995 Yield-line analysis by sequential linear programming. Int. J. Solids Struct. 32, 1395–1404.

[3] Thavalingam, A., Jennings, A., Sloan, D. & McKeown, J. 1999 Computer-assisted generation of yield-line patterns for uniformly loaded isotropic slabs using an optimisation strategy. Engineering Structures 21, 488–496.

Regards

MasterSeries Team ☺